## Further Pure Mathematics FP1 Mark scheme

| Question | Scheme | | Marks |
|---|---|---|---|
| **1** | $\sum_{r=1}^{n} r(r^2-3) = \sum_{r=1}^{n} r^3 - 3\sum_{r=1}^{n} r$ | | |
| | $= \frac{1}{4}n^2(n+1)^2 - 3\left(\frac{1}{2}n(n+1)\right)$ | Attempts to expand $r(r^2-3)$ and attempts to substitute at least one correct standard formula into their resulting expression. | M1 |
| | | Correct expression (or equivalent) | A1 |
| | $= \frac{1}{4}n(n+1)[n(n+1)-6]$ | **dependent on the previous M mark** Attempt to factorise at least $n(n+1)$ having attempted to substitute both the standard formulae | dM1 |
| | $= \frac{1}{4}n(n+1)[n^2+n-6]$ | {this step does not have to be written] | |
| | $= \frac{1}{4}n(n+1)(n+3)(n-2)$ | Correct completion with no errors | A1 cso |
| | | | **(4)** |
| | | | **(4 marks)** |

**Notes:**

Applying eg. $n=1, n=2, n=3$ to the printed equation without applying the standard formulae to give $a=1, b=3, c=-2$ or another combination of these numbers is M0A0M0A0.

__**Alternative Method**__:

Obtains $\sum_{r=1}^{n} r(r^2-3) \equiv \frac{1}{4}n(n+1)[n(n+1)-6] \equiv \frac{1}{4}n(n+a)(n+b)(n+c)$

So $a=1$. $n=1 \Rightarrow -2 = \frac{1}{4}(1)(2)(1+b)(1+c)$ and $n=2 \Rightarrow 0 = \frac{1}{4}(2)(3)(2+b)(2+c)$

leading to either $b=-2, c=3$ or $b=3, c=-2$

**dM1: dependent on the previous M mark.**
Substitutes in values of $n$ and solves to find $b=...$ and $c=...$

**A1:** Finds $a=1, b=3, c=-2$ or another combination of these numbers.

Using **only** a method of "proof by induction" scores 0 marks unless there is use of the standard formulae when the first M1 may be scored.

Allow final dM1A1 for $\frac{1}{4}n^4 + \frac{1}{2}n^3 - \frac{5}{4}n^2 - \frac{3}{2}n$ or $\frac{1}{4}n(n^3+2n^2-5n-6)$

or $\frac{1}{4}(n^4+2n^3-5n^2-6n) \rightarrow \frac{1}{4}n(n+1)(n+3)(n-2)$, from no incorrect working.

Give final A0 for eg. $\frac{1}{4}n(n+1)[n^2+n-6] \rightarrow = \frac{1}{4}n(n+1)(x+3)(x-2)$ unless recovered.

| Question | Scheme | | Marks |
|---|---|---|---|
| 2(a) | $P: y^2 = 28x$ or $P(7t^2, 14t)$ | | |
| | $(y^2 = 4ax \Rightarrow a = 7) \Rightarrow S(7,0)$ | Accept $(7,0)$ or $x = 7, y = 0$ or 7 marked on the $x$-axis in a sketch | B1 |
| | | | **(1)** |
| **(b)** | {$A$ and $B$ have $x$ coordinate} $\frac{7}{2}$ | Divides their $x$ coordinate from (a) by 2 **and** substitutes this into the parabola equation and takes the sqaure root to find $y = ...$ **or applies** $y = \sqrt{\left(2("7") - \left(\frac{"7"}{2}\right)\right)^2 - \left(\frac{"7"}{2}\right)^2}$ **or solves** $7t^2 = 3.5$ and finds $y = 2(7)"\text{their } t"$ | M1 |
| | So $y^2 = 28\left(\frac{7}{2}\right) \Rightarrow y^2 = 98 \Rightarrow y = ...$ **or** $y = \sqrt{(2(7) - 3.5)^2 - (3.5)^2}$ $\left\{= \sqrt{(10.5)^2 - (3.5)^2}\right\}$ **or** $7t^2 = 3.5 \Rightarrow t = \sqrt{0.5} \Rightarrow y = 2(7)\sqrt{0.5}$ | | |
| | $y = (\pm)7\sqrt{2}$ | ***At least one*** correct **exact** value of $y$. Can be un-simplified or simplified. | A1 |
| | $A$, $B$ have coordinates $\left(\frac{7}{2}, 7\sqrt{2}\right)$ and $\left(\frac{7}{2}, -7\sqrt{2}\right)$ | | |
| | Area triangle $ABS =$ | | |
| | • $\frac{1}{2}\left(2(7\sqrt{2})\right)\left(\frac{7}{2}\right)$<br>• $\frac{1}{2}\begin{vmatrix} 7 & 3.5 & 3.5 & 7 \\ 0 & 7\sqrt{2} & -7\sqrt{2} & 0 \end{vmatrix}$ | **dependent on the previous M mark** A **full** method for finding the area of triangle $ABS$. | dM1 |
| | $= \frac{49}{2}\sqrt{2}$ | Correct exact answer. | A1 |
| | | | **(4)** |
| | | | **(5 marks)** |

**Question 2** *continued*

**Notes:**

**(a)**

You can give B1 for part (a) for correct relevant work seen in either part (a) or part (b).

**(b)**

**1st M1:** Allow a slip when candidates find the $x$ coordinate of their midpoint as long as

$0 <$ their midpoint $<$ their $a$

Give 1st M0 if a candidate finds and uses $y = 98$

**1st A1:** Allow any **exact value** of either $7\sqrt{2}, -7\sqrt{2}, \sqrt{98}, -\sqrt{98}, 14\sqrt{0.5}$, awrt 9.9 or awrt $-9.9$

**2nd dM1:** Either $\frac{1}{2}\left(2\times \text{their } "7\sqrt{2}"\right)\left(\text{their } x_{\text{midpoint}}\right)$ or $\frac{1}{2}\left(2\times \text{their } "7\sqrt{2}"\right)\left(\text{their } "7" - x_{\text{midpoint}}\right)$

Condone area triangle $ABS = \left(7\sqrt{2}\right)\left(\frac{7}{2}\right)$, i.e. $\left(\text{their } "7\sqrt{2}"\right)\left(\frac{\text{their } "7"}{2}\right)$

**2nd A1:** Allow exact answers such as $\frac{49}{2}\sqrt{2}, \frac{49}{\sqrt{2}}, 24.5\sqrt{2}, \frac{\sqrt{4802}}{2}, \sqrt{\frac{4802}{4}}, 3.5\sqrt{2}, 49\sqrt{\frac{1}{2}}$

or $\frac{7}{2}\sqrt{98}$ but do not allow $\frac{1}{2}(3.5)\left(2\sqrt{98}\right)$ seen by itself.

Give final A0 for finding 34.64823228… without reference to a correct exact value.

| Question | Scheme | | Marks |
|---|---|---|---|
| **3(a)** | $\mathrm{f}(x)=x^2+\dfrac{3}{x}-1, \quad x<0$ | | |
| | $\mathrm{f}'(x)=2x-3x^{-2}$ | At one of either $x^2 \to \pm Ax$ or $\dfrac{3}{x} \to \pm Bx^{-2}$ where $A$ and $B$ are non-zero constants. | M1 |
| | | Correct differentiation | A1 |
| | $\mathrm{f}(-1.5)=-0.75$, $\mathrm{f}'(-1.5)=-\dfrac{13}{3}$ | Either $\mathrm{f}(-1.5)=-0.75$ or $\mathrm{f}'(-1.5)=-\dfrac{13}{3}$ or awrt $-4.33$ **or a correct numerical expression** for either $\mathrm{f}(-1.5)$ or $\mathrm{f}'(-1.5)$ **Can be implied by later working** | B1 |
| | $\left\{\alpha \simeq -1.5-\dfrac{\mathrm{f}(-1.5)}{\mathrm{f}'(-1.5)}\right\} \Rightarrow \alpha \simeq -1.5-\dfrac{-0.75}{-4.333333...}$ | **dependent on the previous M mark** Valid attempt at Newton-Raphson using their values of $\mathrm{f}(-1.5)$ and $\mathrm{f}'(-1.5)$ | dM1 |
| | $\left\{\alpha=-1.67307692...\text{ or } -\dfrac{87}{52}\right\} \Rightarrow \alpha=-1.67$ | **dependent on all 4 previous marks** $-1.67$ on their first iteration (Ignore any subsequent iterations) | A1 cso cao |
| | **Correct differentiation followed by a correct answer scores full marks in (a)** **Correct answer with <u>no</u> working scores no marks in (a)** | | |
| | | | **(5)** |
| **(b)** | **Way 1** | | |
| | $\mathrm{f}(-1.675)= \quad 0.01458022...$ <br> $\mathrm{f}(-1.665)=-0.0295768...$ | Chooses a suitable interval for $x$, which is within $\pm 0.005$ of their answer to (a) and at least one attempt to evaluate $\mathrm{f}(x)$. | M1 |
| | Sign change (positive, negative) (and $\mathrm{f}(x)$ is continuous) therefore (a root) $\alpha=-1.67$ (2 dp) | Both values correct awrt (or truncated) 1 sf, sign change and conclusion. | A1 cso |
| | | | **(2)** |

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="5">3(b)<br>continued</td><td colspan="3">Way 2</td></tr>
<tr><td colspan="2">Alt 1: Applying Newton-Raphson again Eg. Using $\alpha = -1.67, -1.673$ or $-\frac{87}{52}$</td><td></td></tr>
<tr><td>• $\alpha \simeq -1.67 - \frac{-0.007507185629...}{-4.415692926...}$ $\{= -1.671700115...\}$<br>• $\alpha \simeq -1.673 - \frac{0.005743106396...}{-4.41783855...}$ $\{= -1.671700019...\}$<br>• $\alpha \simeq -\frac{87}{52} - \frac{0.006082942257...}{-4.417893838...}$ $\{= -1.67170036...\}$</td><td>Evidence of applying Newton-Raphson for a second time on their answer to part (a)</td><td>M1</td></tr>
<tr><td>So $\alpha = -1.67$ (2 dp)</td><td>$\alpha = -1.67$</td><td>A1</td></tr>
<tr><td colspan="2"></td><td>(2)</td></tr>
<tr><td colspan="4" align="right">(7 marks)</td></tr>
</table>

**Notes:**

**(a)**

Incorrect differentiation followed by their estimate of $\alpha$ with no evidence of applying the NR formula is final dM0A0.

**B1:** B1 can be given for a correct numerical expression for either $f(-1.5)$ or $f'(-1.5)$

Eg. either $(-1.5)^2 + \frac{3}{(-1.5)} - 1$ or $2(-1.5) - \frac{3}{(-1.5)^2}$ are fine for B1.

Final -This mark can be implied by applying at least one correct value of either $f(-1.5)$ or $f'(-1.5)$

**dM1:** in $-1.5 - \frac{f(-1.5)}{f'(-1.5)}$. So just $-1.5 - \frac{f(-1.5)}{f'(-1.5)}$ with an incorrect answer and no other evidence scores final dM0A0.

Give final dM0 for applying $1.5 - \frac{f(-1.5)}{f'(-1.5)}$ without first quoting the correct N-R formula.

**(b)**

**A1:** **Way 1:** correct solution only

Candidate needs to state both of their values for $f(x)$ to awrt (or truncated) 1 sf along with a reason and conclusion. Reference to change of sign or eg. $f(-1.675) \times f(-1.665) < 0$ or a diagram or < 0 and > 0 or one positive, one negative are sufficient reasons. There must be a (minimal, not incorrect) conclusion, eg. $\alpha = -1.67$, root (or $\alpha$ or part (a)) is correct, QED and a square are all acceptable. Ignore the presence or absence of any reference to continuity.

A minimal acceptable reason and conclusion is "change of sign, hence root".

No explicit reference to 2 decimal places is required.

Stating "root is in between – 1.675 and – 1.665" without some reference to is not sufficient for A1

Accept 0.015 as a correct evaluation of $f(-1.675)$

**Question 3 notes** *continued*

**(b)**

**A1:** **Way 2:** correct solution only

Their conclusion in Way 2 needs to convey that they understand that $\alpha = -1.67$ to 2 decimal places. Eg. "therefore my answer to part (a) [which must be $-1.67$] is correct" is fine for A1.

$-1.67 - \dfrac{\mathrm{f}(-1.67)}{\mathrm{f}'(1.67)} = -1.67$ (2 dp) is sufficient for M1A1 in part (b).

The root of $\mathrm{f}(x) = 0$ is $-1.67169988...$, so candidates can also choose $x_1$ which is less than $-1.67169988...$ and choose $x_2$ which is greater than $-1.67169988...$ with both $x_1$ and $x_2$ lying in the interval $[-1.675, -1.665]$ and evaluate $\mathrm{f}(x_1)$ and $\mathrm{f}(x_2)$.

**Helpful Table**

| $x$ | $\mathrm{f}(x)$ |
|---|---|
| $-1.675$ | 0.014580224 |
| $-1.674$ | 0.010161305 |
| $-1.673$ | 0.005743106 |
| $-1.672$ | 0.001325627 |
| $-1.671$ | $-0.003091136$ |
| $-1.670$ | $-0.007507186$ |
| $-1.669$ | $-0.011922523$ |
| $-1.668$ | $-0.016337151$ |
| $-1.667$ | $-0.020751072$ |
| $-1.666$ | $-0.025164288$ |
| $-1.665$ | $-0.029576802$ |

| Question | Scheme | | Marks |
|---|---|---|---|
| **4(a)** | $\mathbf{A} = \begin{pmatrix} k & 3 \\ -1 & k+2 \end{pmatrix}$ where $k$ is a constant and let $g(k) = k^2 + 2k + 3$ | | |
| | $\{\det(\mathbf{A}) =\}\ k(k+2)+3$ or $k^2+2k+3$ | Correct $\det(\mathbf{A})$, un-simplified or simplified | B1 |
| | **Way 1** | | |
| | $= (k+1)^2 - 1 + 3$ | Attempts to complete the square [usual rules apply] | M1 |
| | $= (k+1)^2 + 2 > 0$ | $(k+1)^2 + 2$ and $> 0$ | A1 cso |
| | | | **(3)** |
| | **Way 2** | | |
| | $\{\det(\mathbf{A}) =\}\ k(k+2)+3$ or $k^2+2k+3$ | Correct $\det(\mathbf{A})$, un-simplified or simplified | B1 |
| | $\{b^2 - 4ac =\}\ 2^2 - 4(1)(3)$ | Applies "$b^2 - 4ac$" to their $\det(\mathbf{A})$ | M1 |
| | **All of**<br>• $b^2 - 4ac = -8 < 0$<br>• some reference to $k^2 + 2k + 3$ being above the $x$-axis<br>• so $\det(\mathbf{A}) > 0$ | Complete solution | A1 cso |
| | | | **(3)** |
| | **Way 3** | | |
| | $\{g(k) = \det(\mathbf{A}) =\}\ k(k+2)+3$ or $k^2+2k+3$ | Correct $\det(\mathbf{A})$, un-simplified or simplified | B1 |
| | $g'(k) = 2k + 2 = 0 \Rightarrow k = -1$<br>$g_{\min} = (-1)^2 + 2(-1) + 3$ | Finds the value of $k$ for which $g'(k) = 0$ **and** substitutes this value of $k$ into $g(k)$ | M1 |
| | $g_{\min} = 2$, so $\det(\mathbf{A}) > 0$ | $g_{\min} = 2$ and states $\det(\mathbf{A}) > 0$ | A1 cso |
| | | | **(3)** |
| **(b)** | $\mathbf{A}^{-1} = \dfrac{1}{k^2 + 2k + 3}\begin{pmatrix} k+2 & -3 \\ 1 & k \end{pmatrix}$ | $\dfrac{1}{\text{their } \det(\mathbf{A})}\begin{pmatrix} k+2 & -3 \\ 1 & k \end{pmatrix}$ | M1 |
| | | Correct answer in terms of $k$ | A1 |
| | | | **(2)** |
| | | | **(5 marks)** |

| **Question 4** *continued* | |
|---|---|
| **Notes:** | |
| **(a)** | |
| **B1:** | Also allow $k(k+2)--3$ |
| | **Way 2:** Proving $b^2-4ac=-8<0$ by itself could mean that $\det(\mathbf{A})>0$ or $\det(\mathbf{A})<0$. |
| | To gain the final A1 mark for Way 2, candidates need to show $b^2-4ac=-8<0$ **and** make some reference to $k^2+2k+3$ being above the $x$-axis (eg. states that coefficient of $k^2$ is positive **or** evaluates $\det(\mathbf{A})$ for any value of $k$ to give a positive result **or** sketches a quadratic curve that is above the $x$-axis) before then stating that $\det(\mathbf{A})>0$. |
| | Attempting to solve $\det(\mathbf{A})=0$ by applying the quadratic formula or finding $-1\pm\sqrt{2}\mathrm{i}$ is enough to score the M1 mark for Way 2. To gain A1 these candidates need to make some reference to $k^2+2k+3$ being above the $x$-axis (eg. states that coefficient of $k^2$ is positive **or** evaluates $\det(\mathbf{A})$ for any value of $k$ to give a positive result **or** sketches a quadratic curve that is above the $x$-axis) before then stating that $\det(\mathbf{A})>0$. |
| **(b)** | |
| **A1:** | Allow either $\frac{1}{(k+1)^2+2}\begin{pmatrix} k+2 & -3 \\ 1 & k \end{pmatrix}$ or $\begin{pmatrix} \frac{k+2}{k^2+2k+3} & \frac{-3}{k^2+2k+3} \\ \frac{1}{k^2+2k+3} & \frac{k}{k^2+2k+3} \end{pmatrix}$ or equivalent. |

| Question | Scheme | | Marks |
|---|---|---|---|
| 5 | $2z + z^* = \dfrac{3+4\text{i}}{7+\text{i}}$ | | |
| | **Way 1** | | |
| | $\{2z + z^* =\}\ 2(a+\text{i}b) + (a-\text{i}b)$ | Left hand side = $2(a+\text{i}b) + (a-\text{i}b)$<br>Can be implied by eg. $3a + \text{i}b$<br>**Note:** This can be seen anywhere in their solution | B1 |
| | $\ldots\ldots\ldots = \dfrac{(3+4\text{i})}{(7+\text{i})}\dfrac{(7-\text{i})}{(7-\text{i})}$ | Multiplies numerator **and** denominator of the right hand side by $7-\text{i}$ or $-7+\text{i}$ | M1 |
| | $\ldots\ldots\ldots = \dfrac{25+25\text{i}}{50}$ | Applies $\text{i}^2 = -1$ to and collects like terms to give<br>right hand side = $\dfrac{25+25\text{i}}{50}$ or equivalent | A1 |
| | So, $3a + \text{i}b = \dfrac{1}{2} + \dfrac{1}{2}\text{i}$<br>$\Rightarrow a = \dfrac{1}{6}, b = \dfrac{1}{2}$ **or** $z = \dfrac{1}{6} + \dfrac{1}{2}\text{i}$ | **dependent on the previous B and M marks**<br>Equates either real parts or imaginary parts to give at least one of $a = \ldots$ or $b = \ldots$ | ddM1 |
| | | Either $a = \dfrac{1}{6}$ **and** $b = \dfrac{1}{2}$ **or** $z = \dfrac{1}{6} + \dfrac{1}{2}\text{i}$ | A1 |
| | | | **(5)** |
| | **Way 2** | | |
| | $\{2z + z^* =\}\ 2(a+\text{i}b) + (a-\text{i}b)$ | Left hand side = $2(a+\text{i}b) + (a-\text{i}b)$<br>Can be implied by eg. $3a + \text{i}b$ | B1 |
| | $(3a + \text{i}b)(7+\text{i}) = \ldots\ldots\ldots$ | Multiplies their $(3a + \text{i}b)$ by $(7+\text{i})$ | M1 |
| | $21a + 3a\text{i} + 7b\text{i} - b = \ldots\ldots\ldots$ | Applies $\text{i}^2 = -1$ to give<br>left hand side = $21a + 3a\text{i} + 7b\text{i} - b$ | A1 |
| | So, $(21a - b) + (3a + 7b) = 3 + 4\text{i}$<br>gives $21a - b = 3$, $3a + 7b = 4$<br>$\Rightarrow a = \dfrac{1}{6}, b = \dfrac{1}{2}$ **or** $z = \dfrac{1}{6} + \dfrac{1}{2}\text{i}$ | **dependent on the previous B and M marks**<br>Equates **both** real parts and imaginary parts to give at least one of $a = \ldots$ or $b = \ldots$ | ddM1 |
| | | Either $a = \dfrac{1}{6}$ **and** $b = \dfrac{1}{2}$ **or** $z = \dfrac{1}{6} + \dfrac{1}{2}\text{i}$ | A1 |
| | | | **(5)** |
| | | | **(5 marks)** |

| **Question 5** *continued* |
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| **Notes:** |
| Some candidates may let $z = x + \mathrm{i}y$ and $z^* = x - \mathrm{i}y$.<br>So apply the mark scheme with $x \equiv a$ and $y \equiv b$.<br>For the final A1 mark, you can accept exact equivalents for $a$, $b$. |

| Question | Scheme | | Marks |
|---|---|---|---|
| 6(a) | $H: xy = 25$, $P\left(5t, \frac{5}{t}\right)$ is a general point on $H$ | | |
| | Either $5t\left(\frac{5}{t}\right) = 25$ **or** $y = \frac{25}{x} = \frac{25}{5t} = \frac{5}{t}$ **or** $x = \frac{25}{y} = \frac{25}{\frac{5}{t}} = 5t$ **or** states $c = 5$ | | B1 |
| | | | **(1)** |
| (b) | $y = \frac{25}{x} = 25x^{-1} \Rightarrow \frac{dy}{dx} = -25x^{-2} = -\frac{25}{x^2}$ | $\frac{dy}{dx} = \pm k x^{-2}$ where $k$ is a numerical value | M1 |
| | $xy = 25 \Rightarrow x\frac{dy}{dx} + y = 0$ | Correct use of product rule. The sum of two terms, one of which is correct. | |
| | $\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = -\frac{5}{t^2}\left(\frac{1}{5}\right)$ | $\frac{dy}{dt} \times \frac{1}{\text{their } \frac{dx}{dt}}$ | |
| | $\left\{\text{At } A,\ t = \frac{1}{2},\ x = \frac{5}{2},\ y = 10\right\} \Rightarrow \frac{dy}{dx} = -4$ | Correct numerical gradient at $A$, which is found using calculus. **Can be implied by later working** | A1 |
| | So, $m_N = \frac{1}{4}$ | Applies $m_N = \frac{-1}{m_T}$, to find a numerical $m_N$, where $m_T$ is found from using calculus. **Can be implied by later working** | M1 |
| | • $y - 10 = \frac{1}{4}\left(x - \frac{5}{2}\right)$<br>• $10 = \frac{1}{4}\left(\frac{5}{2}\right) + c \Rightarrow c = \frac{75}{8} \Rightarrow y = \frac{1}{4}x + \frac{75}{8}$ | Correct line method for a **normal** where a numerical $m_N (\neq m_T)$ is found from using calculus. **Can be implied by later working** | M1 |
| | leading to $8y - 2x - 75 = 0$ (*) | Correct solution only | A1 |
| | | | **(5)** |

| Question | Scheme | | Marks |
|---|---|---|---|
| 6(c) | $y=\frac{25}{x} \Rightarrow 8\left(\frac{25}{x}\right)-2x-75=0$ **or** $x=\frac{25}{y} \Rightarrow 8y-2\left(\frac{25}{y}\right)-75=0$<br>**or** $x=5t,\ y=\frac{5}{t} \Rightarrow 8(5t)-2\left(\frac{5}{t}\right)-75=0$<br>Substitutes $y=\frac{25}{x}$ **or** $x=\frac{25}{y}$ **or** $x=5t$ **and** $y=\frac{5}{t}$ into the printed equation or their normal equation to obtain an equation in either $x$ only, $y$ only or $t$ only | | M1 |
| | $2x^2+75x-200=0$ **or** $8y^2-75y-50=0$ **or** $2t^2+15t-8=0$ **or** $10t^2+75t-40=0$ | | |
| | $(2x-5)(x+40)=0 \Rightarrow x=...$ **or** $(y-10)(8y+5)=0 \Rightarrow y=...$ **or** $(2t-1)(t+8)=0 \Rightarrow t=...$<br>**dependent on the previous M mark**<br>Correct attempt of solving a 3TQ to find either $x=...,\ y=...$ or $t=...$ | | dM1 |
| | Finds at least one of either $x=-40$ or $y=-\frac{5}{8}$ | | A1 |
| | $B\left(-40,\ -\frac{5}{8}\right)$ | Both correct coordinates (If coordinates are not stated they can be paired together as $x=...,\ y=...$ ) | A1 |
| | | | **(4)** |
| | | | **(10 marks)** |

**Notes:**

**(a)** A conclusion is not required on this occasion in part (a).

**B1:** Condone reference to $c=5$ (as $xy=c^2$ and $\left(ct,\frac{c}{t}\right)$ are referred in the Formula book.)

**(b)**

$\frac{dy}{dx}=\frac{dy}{dt}.\frac{dt}{dx}=-\frac{5}{t^2}\left(\frac{1}{5}\right)=-\frac{1}{t^2} \Rightarrow m_N=t^2 \Rightarrow y-10=t^2\left(x-\frac{5}{2}\right)$ **scores only the first M1.**

When $t=\frac{1}{2}$ is substituted giving $y-10=\frac{1}{4}\left(x-\frac{5}{2}\right)$ the response then automatically gets A1(implied) M1(implied) M1

| **Question 6 notes** *continued* |
| --- |
| **(c)**<br>You can imply the final three marks (dM1A1A1) for either<br>• $8\left(\frac{25}{x}\right) - 2x - 75 = 0 \rightarrow \left(-40, -\frac{5}{8}\right)$<br>• $8y - 2\left(\frac{25}{y}\right) - 75 = 0 \rightarrow \left(-40, -\frac{5}{8}\right)$<br>• $8(5t) - 2\left(\frac{5}{t}\right) - 75 = 0 \rightarrow \left(-40, -\frac{5}{8}\right)$<br>with no intermediate working.<br><br>You can also imply the middle dM1A1 marks for either<br>• $8\left(\frac{25}{x}\right) - 2x - 75 = 0 \rightarrow x = -40$<br>• $8y - 2\left(\frac{25}{y}\right) - 75 = 0 \rightarrow y = -\frac{5}{8}$<br>• $8(5t) - 2\left(\frac{5}{t}\right) - 75 = 0 \rightarrow x = -40$ or $y = -\frac{5}{8}$<br>with no intermediate working.<br><br>**Writing** $x = -40,\ y = -\frac{5}{8}$ followed by $B\left(40, \frac{5}{8}\right)$ or $B\left(-\frac{5}{8}, -40\right)$ is final A0.<br><br>Ignore stating $B\left(\frac{5}{2}, 10\right)$ in addition to $B\left(-40, -\frac{5}{8}\right)$ |

| Question | Scheme | | Marks |
|---|---|---|---|
| **7(a)** | Rotation | Rotation | B1 |
| | 67 degrees (anticlockwise) | Either $\arctan\left(\frac{12}{5}\right)$, $\tan^{-1}\left(\frac{12}{5}\right)$, $\sin^{-1}\left(\frac{12}{13}\right)$, $\cos^{-1}\left(\frac{5}{13}\right)$, awrt 67 degrees, awrt 1.2, truncated 1.1 (anticlockwise), awrt 293 degrees clockwise or awrt 5.1 clockwise | B1 o.e. |
| | about $(0, 0)$ | **The mark is dependent on at least one of the previous B marks being awarded.** About $(0, 0)$ or about $O$ or about the origin | dB1 |
| | **Note:** Give 2nd B0 for 67 degrees clockwise o.e. | | **(3)** |
| **(b)** | $\{\mathbf{Q} =\}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ | Correct matrix | B1 |
| | | | **(1)** |
| **(c)** | $\{\mathbf{R} = \mathbf{PQ}\}\begin{pmatrix} \frac{5}{13} & -\frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; = \begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix}$ | Multiplies **P** by their **Q** in the correct order and finds at least one element | M1 |
| | | Correct matrix | A1 |
| | | | **(2)** |
| **(d)** | **Way 1** | | |
| | $\begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix}\begin{pmatrix} x \\ kx \end{pmatrix} = \begin{pmatrix} x \\ kx \end{pmatrix}$ | Forming the equation "their matrix $\mathbf{R}$" $\begin{pmatrix} x \\ kx \end{pmatrix} = \begin{pmatrix} x \\ kx \end{pmatrix}$ **Allow $x$ being replaced by any non-zero number eg. 1.** Can be implied by at least one correct ft equations below. | M1 |
| | $-\frac{12}{13}x + \frac{5kx}{13} = x$ or $\frac{5}{13}x + \frac{12kx}{13} = kx \Rightarrow k = ...$ | Uses their matrix equation to form an equation in $k$ and progresses to give $k =$ numerical value | M1 |
| | So $k = 5$ | **dependent on only the previous M mark** $k = 5$ | A1 **cao** |
| | **Dependent on all previous marks being scored in this part.** Either<br>• Solves **both** $-\frac{12}{13}x + \frac{5kx}{13} = x$ **and** $\frac{5}{13}x + \frac{12kx}{13} = kx$ to give $k = 5$<br>• Finds $k = 5$ and checks that it is true for the other component<br>• Confirms that $\begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix}\begin{pmatrix} x \\ 5x \end{pmatrix} = \begin{pmatrix} x \\ 5x \end{pmatrix}$ | | A1 **cso** |
| | | | **(4)** |

| Question | Scheme | | Marks |
|---|---|---|---|
| 7(d) *continued* | **Way 2** | | |
| | Either $\cos 2\theta = -\frac{12}{13}, \sin 2\theta = \frac{5}{13}$ or $\tan 2\theta = -\frac{5}{12}$ | Correct follow through equation in $2\theta$ based on their matrix $\mathbf{R}$ | M1 |
| | | Full method of finding $2\theta$, then $\theta$ and applying $\tan\theta$ | M1 |
| | $\{k =\}\tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ | $\tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ or $\tan(\text{awrt } 78.7^\circ)$ or $\tan(\text{awrt } 1.37)$. **Can be implied.** | A1 |
| | So $k = 5$ | $k = 5$ **by a correct solution only** | A1 |
| | | | **(4)** |
| | | | **(10 marks)** |

**Notes:**

**(a)**

Condone "Turn" for the 1st B1 mark.

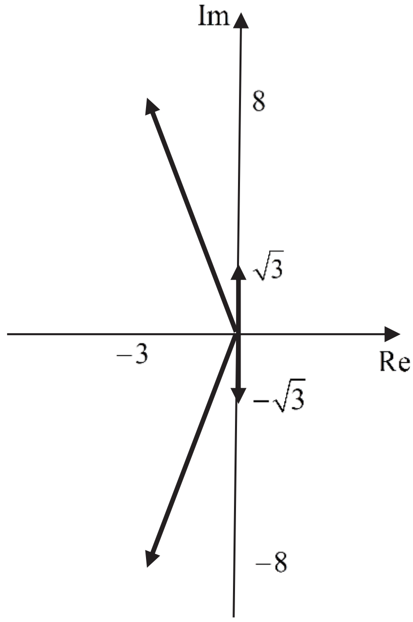
Penalise the first B1 mark for candidates giving a combination of transformations.

**(c)**

Allow 1st M1 for eg. $\text{"their matrix } \mathbf{R}\text{"}\begin{pmatrix}1\\k\end{pmatrix} = \begin{pmatrix}1\\k\end{pmatrix}$ or $\text{"their matrix } \mathbf{R}\text{"}\begin{pmatrix}k\\k^2\end{pmatrix} = \begin{pmatrix}k\\k^2\end{pmatrix}$

or $\text{"their matrix } \mathbf{R}\text{"}\begin{pmatrix}\frac{1}{k}\\1\end{pmatrix} = \begin{pmatrix}\frac{1}{k}\\1\end{pmatrix}$ or equivalent

$$y = (\tan\theta)x : \begin{pmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{pmatrix} = \begin{pmatrix}-\frac{12}{13} & \frac{5}{13}\\ \frac{5}{13} & \frac{12}{13}\end{pmatrix}$$

| Question | Scheme | | Marks |
|---|---|---|---|
| **8(a)** | $\mathrm{f}(z) = z^4 + 6z^3 + 76z^2 + az + b$, $a, b$ are real constants. $z_1 = -3 + 8\mathrm{i}$ is given. | | |
| | $-3-8\mathrm{i}$ | $-3-8\mathrm{i}$ | B1 |
| | | | **(1)** |
| **(b)** | $z^2 + 6z + 73$ | Attempt to expand $(z-(-3+8\mathrm{i}))(z-(-3-8\mathrm{i}))$ or any valid method ***to establish a quadratic factor*** eg $z = -3 \pm 8\mathrm{i} \Rightarrow z+3 = \pm 8\mathrm{i} \Rightarrow z^2+6z+9 = -64$ or sum of roots $-6$, product of roots 73 to give $z^2 \pm (\text{sum})z + \text{product}$ | M1 |
| | | $z^2 + 6z + 73$ | A1 |
| | $\mathrm{f}(z) = (z^2+6z+73)(z^2+3)$ | Attempts to find the other quadratic factor. eg. using long division to get as far as $z^2 + ...$ or eg. $\mathrm{f}(z) = (z^2+6z+73)(z^2+...)$ | M1 |
| | | $z^2 + 3$ | A1 |
| | $\left\{z^2+3=0 \Rightarrow z=\right\}$ $\pm\sqrt{3}\,\mathrm{i}$ | **dependent on only the previous M mark** Correct method of solving the 2nd quadratic factor | dM1 |
| | | $\sqrt{3}\,\mathrm{i}$ **and** $-\sqrt{3}\,\mathrm{i}$ | A1 |
| | | | **(6)** |
| **(c)** |  | <u>**Criteria**</u><br>• $-3\pm 8\mathrm{i}$ plotted correctly in quadrants 2 and 3 with some evidence of symmetry<br>• Their other two ***complex roots*** (which are found from solving their 2nd quadratic in (b)) are plotted correctly with some evidence of symmetry about the $x$-axis | |
| | | Satisfies at least one of the two criteria | B1 ft |
| | | Satisfies both criteria with some indication of scale or coordinates stated. All points (arrows) must be in the correct positions relative to each other. | B1 ft |
| | | | **(2)** |
| | | | **(9 marks)** |

| **Question 8** *continued* |
| --- |
| **Notes:** |
| **(b)**<br>Give 3rd M1 for $z^2 + k = 0,\ k > 0 \Rightarrow$ **at least one of either** $z = \sqrt{k}\,\mathrm{i}$ **or** $z = -\sqrt{k}\,\mathrm{i}$<br>Give 3rd M0 for $z^2 + k = 0,\ k > 0 \Rightarrow z = \pm k\mathrm{i}$<br>Give 3rd M0 for $z^2 + k = 0,\ k > 0 \Rightarrow z = \pm k$ or $z = \pm\sqrt{k}$<br>Candidates do not need to find $a = 18,\ b = 219$ |

| Question | Scheme | | Marks |
|---|---|---|---|
| 9(a) | $2x^2+4x-3=0$ has roots $\alpha$, $\beta$ | | |
| | $\alpha+\beta=-\frac{4}{2}$ or $-2$, $\alpha\beta=-\frac{3}{2}$ | **Both** $\alpha+\beta=-\frac{4}{2}$ **and** $\alpha\beta=-\frac{3}{2}$. This may be seen or implied anywhere in this question. | B1 |
| (i) | $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=......$ | **Use** of a **correct** identity for $\alpha^2+\beta^2$ (May be implied by their work) | M1 |
| | $=(-2)^2-2\left(-\frac{3}{2}\right)=7$ | 7 **from correct working** | A1 cso |
| (ii) | $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=......$<br>or $=(\alpha+\beta)\left(\alpha^2+\beta^2-\alpha\beta\right)=......$ | **Use** of an **appropriate** and **correct** identity for $\alpha^3+\beta^3$ (May be implied by their work) | M1 |
| | $=(-2)^3-3\left(-\frac{3}{2}\right)(-2)=-17$<br>or $=(-2)\left(7--\frac{3}{2}\right)=-17$ | $-17$ **from correct working** | A1 cso |
| | | | **(5)** |
| (b) | Sum $=\alpha^2+\beta+\beta^2+\alpha$<br>$=\alpha^2+\beta^2+\alpha+\beta$<br>$=7+(-2)=5$ | Uses at least one of their $\alpha^2+\beta^2$ or $\alpha+\beta$ in an attempt to find a **numerical value** for the sum of $(\alpha^2+\beta)$ and $(\beta^2+\alpha)$ | M1 |
| | Product $=(\alpha^2+\beta)(\beta^2+\alpha)$<br>$=(\alpha\beta)^2+\alpha^3+\beta^3+\alpha\beta$<br>$=\left(-\frac{3}{2}\right)^2-17-\frac{3}{2}=-\frac{65}{4}$ | Expands $(\alpha^2+\beta)(\beta^2+\alpha)$ and uses at least one of their $\alpha\beta$ or $\alpha^3+\beta^3$ in an attempt to find a **numerical value** for the product of $(\alpha^2+\beta)$ and $(\beta^2+\alpha)$ | M1 |
| | $x^2-5x-\frac{65}{4}=0$ | Applies $x^2-(\text{sum})x+\text{product}$ (Can be implied) (" = 0" not required) | M1 |
| | $4x^2-20x-65=0$ | Any integer multiple of $4x^2-20x-65=0$, including the "= 0" | A1 |
| | | | **(4)** |

| Question | Scheme | | Marks |
|---|---|---|---|
| **9(b)** *continued* | **Alternative:** Finding $\alpha^2+\beta$ and $\beta^2+\alpha$ explicitly | | |
| | Eg. Let $\alpha=\frac{-4+\sqrt{40}}{4}$, $\beta=\frac{-4+\sqrt{40}}{4}$ and so $\alpha^2+\beta=\frac{5-3\sqrt{10}}{2}$, $\beta^2+\alpha=\frac{5+3\sqrt{10}}{2}$ | | |
| | $\left(x-\left(\frac{5-3\sqrt{10}}{2}\right)\right)\left(x-\left(\frac{5+3\sqrt{10}}{2}\right)\right)$ | Uses $\left(x-\left(\alpha^2+\beta\right)\right)\left(x-\left(\beta^2+\alpha\right)\right)$ with exact numerical values. (May expand first) | M1 |
| | $=x^2-\left(\frac{5+3\sqrt{10}}{2}\right)x-\left(\frac{5-3\sqrt{10}}{2}\right)x+\left(\frac{5-3\sqrt{10}}{2}\right)\left(\frac{5+3\sqrt{10}}{2}\right)$ | Attempts to expand using exact numerical values for $\alpha^2+\beta$ and $\beta^2+\alpha$ | M1 |
| | $\Rightarrow x^2-5x-\frac{65}{4}=0$ | Collect terms to give a 3TQ. ("$=0$" not required) | M1 |
| | $4x^2-20x-65=0$ | Any integer multiple of $4x^2-20x-65=0$, including the "= 0" | A1 |
| | | | **(4)** |
| | | | **(9 marks)** |

**Notes:**

**(a)**

**1st A1:** $\alpha+\beta=2,\ \alpha\beta=-\frac{3}{2}\Rightarrow\alpha^2+\beta^2=4-2\left(-\frac{3}{2}\right)=7$ is M1A0 cso

Finding $\alpha+\beta=-2,\alpha\beta=-\frac{3}{2}$ by writing down or applying $\frac{-4+\sqrt{40}}{4},\frac{-4+\sqrt{40}}{4}$ but then writing $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=4+3=7$ and $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=-8-9=-17$ scores B0M1A0M1A0 in part (a).

Applying $\frac{-4+\sqrt{40}}{4},\frac{-4+\sqrt{40}}{4}$ explicitly in part (a) will score B0M0A0M0A0

Eg: Give no credit for $\left(\frac{-4+\sqrt{40}}{4}\right)^2+\left(\frac{-4+\sqrt{40}}{4}\right)^2=7$

or for $\left(\frac{-4+\sqrt{40}}{4}\right)^3+\left(\frac{-4+\sqrt{40}}{4}\right)^3=-17$

**(b)**

Candidates **are allowed** to apply $\frac{-4+\sqrt{40}}{4},\frac{-4+\sqrt{40}}{4}$ explicitly in part (b).

A correct method leading to a candidate stating $a=4, b=-20, c=-65$ without writing a final answer of $4x^2-20x-65=0$ is **final** M1A0

| Question | Scheme | | Marks |
|---|---|---|---|
| **10** | $u_1 = 5,\ u_{n+1} = 3u_n + 2,\ n \geq 1$. Required to prove the result, $u_n = 2\times(3)^n - 1,\ n \in \mathbb{Z}^+$ | | |
| **(i)** | $n=1:\ u_1 = 2(3) - 1 = 5$ | $u_1 = 2(3) - 1 = 5$ or $u_1 = 6 - 1 = 5$ | B1 |
| | (Assume the result is true for $n = k$) | | |
| | $u_{k+1} = 3\left(2(3)^k - 1\right) + 2$ | Substitutes $u_k = 2(3)^k - 1$ into $u_{k+1} = 3u_k + 2$ | M1 |
| | $= 2(3)^{k+1} - 1$ | **dependent on the previous M mark** Expresses $u_{k+1}$ in term of $3^{k+1}$ | dM1 |
| | | $u_{k+1} = 2(3)^{k+1} - 1$ **by correct solution only** | A1 |
| | If the result is <u>true for $n = k$,</u> then it is <u>true for $n = k+1$.</u> As the result has been shown to be <u>true for $n = 1$,</u> then the result <u>is true for all $n$</u> | | A1 cso |
| | | | **(5)** |
| | Required to prove the result $\sum_{r=1}^{n} \frac{4r}{3^r} = 3 - \frac{(3+2n)}{3^n}$ , $n \in \mathbb{Z}^+$ | | |
| **(ii)** | $n = 1:\text{LHS} = \frac{4}{3}$ , $\text{RHS} = 3 - \frac{5}{3} = \frac{4}{3}$ | Shows or states **both** $\text{LHS} = \frac{4}{3}$ **and** $\text{RHS} = \frac{4}{3}$ **or** states $\text{LHS} = \text{RHS} = \frac{4}{3}$ | B1 |
| | (Assume the result is true for $n = k$ ) | | |
| | $\sum_{r=1}^{k+1} \frac{4r}{3^r} = 3 - \frac{(3+2k)}{3^k} + \frac{4(k+1)}{3^{k+1}}$ | Adds the $(k+1)^{\text{th}}$ term to the sum of $k$ terms | M1 |
| | $= 3 - \frac{3(3+2k)}{3^{k+1}} + \frac{4(k+1)}{3^{k+1}}$ | **dependent on the previous M mark** Makes $3^{k+1}$ or $(3)3^k$ a common denominator for their fractions. | dM1 |
| | | Correct expression with common denominator $3^{k+1}$ or $(3)3^k$ for their fractions. | A1 |
| | $= 3 - \left(\frac{3(3+2k) - 4(k+1)}{3^{k+1}}\right)$ <br> $= 3 - \left(\frac{5+2k}{3^{k+1}}\right)$ | | |
| | $= 3 - \frac{(3+2(k+1))}{3^{k+1}}$ | $3 - \frac{(3+2(k+1))}{3^{k+1}}$ **by correct solution only** | A1 |
| | If the result is <u>true for $n = k$,</u> then it is <u>true for $n = k+1$.</u> As the result has been shown to be <u>true for $n = 1$,</u> then the result <u>is true for all $n$</u> | | A1 cso |
| | | | **(6)** |
| | | | **(11 marks)** |

| |
|---|
| **Question 10** *continued* |
| **Notes:** |
| **(i) & (ii)**<br>**Final A1 for parts (i) and (ii)** is dependent on all previous marks being scored in that part.<br>It is gained by candidates conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution. |
| **(i)**<br>$u_1 = 5$ by itself is not sufficient for the 1st B1 mark in part (i).<br>$u_1 = 3 + 2$ without stating $u_1 = 2(3) - 1 = 5$ or $u_1 = 6 - 1 = 5$ is B0 |
| **(ii)**<br>LHS = RHS by itself is not sufficient for the 1st B1 mark in part (ii). |