Mechanics M3 Mark scheme

Question	Scheme	Marks
1	(30^{o} or θ for the first 3 lines)	
	$R\sin 30° = mg$	M1 A1
	$R\cos 30° = m(r\cos 30°)\omega^2$	M1 A1 A1
	$\omega^2 = \frac{R}{mr} = \frac{g}{r\sin 30}$	DM1
	$\omega = \sqrt{\frac{2g}{r}}$	A1
	Time = $\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{2g}} = \pi\sqrt{\frac{2r}{g}}$ *	A1 cso
		(8)
	Alternative:	
	Resolve perpendicular to the reaction:	
	$mg\cos 30 = m \times rad \times \omega^2 \cos 60$	M2 A1 (LHS) A1 (RHS)
	$= mr\cos 30\omega^2 \cos 60$	A1
	Obtain ω	M1 A1
	Correct time	A1
		(8)
		(8 marks)

Notes:

M1: Resolving vertically 30^{o} or θ

A1: Correct equation 30^{o} or θ

M1: Attempting an equation of motion along the radius, acceleration in either form 30^{o} or θ Allow with r for radius.

A1: LHS correct 30^{o} or θ

A1: RHS correct, 30^{o} or θ but not r for radius.

DM1: Obtaining an expression for ω^2 or for v^2 **and** the length of the path 30^{o} or θ Dependent on both previous M marks.

A1: Correct expression for ω Must have the numerical value for the trig function now.

A1cso: Deducing the GIVEN answer.

Question	Scheme	Marks
2(a)	$F = \frac{K}{x^2}$	
	$x = R \Rightarrow F = mg \quad \therefore mg = \frac{K}{R^2}$	M1
	$K = mgR^2$ *	A1
		(2)
(b)	$\frac{mgR^2}{x^2} = -mv\frac{\mathrm{d}v}{\mathrm{d}x}$	M1
	$g\int \frac{R^2}{x^2}\,\mathrm{d}x = -\int v\,\mathrm{d}v$	
	$-g\frac{R^2}{x} = -\frac{1}{2}v^2 \quad (+c)$	dM1 A1ft
	$x = 3R,\ v = V \Rightarrow -g\frac{R^2}{3R} = -\frac{1}{2}V^2 + c$	M1
	$c = -\frac{Rg}{3} + \frac{1}{2}V^2$	A1
	$x = R \Rightarrow \frac{1}{2}v^2 = -\frac{Rg}{3} + \frac{1}{2}V^2 + g\frac{R^2}{R}$	M1
	$v^2 = V^2 + \frac{4Rg}{3}$	
	$v = \sqrt{V^2 + \frac{4Rg}{3}}$	A1 cso
		(7)
		(9 marks)

Notes:

(a)

M1: Setting $F = mg$ **and** $x = R$

A1: Deducing the GIVEN answer

(b)

M1: Attempting an equation of motion with acceleration in the form $v\frac{\mathrm{d}v}{\mathrm{d}x}$. The minus sign may be missing.

dM1: Attempting the integration.

A1ft: Correct integration, follow through on a missing minus sign from line 1, constant of integration may be missing.

M1: Substituting $x = 3R, v = V$ to obtain an equation for c

A1: Correct expression for c.

M1: Substituting $x = R$ and their expression for c.

A1: Correct expression for v, any equivalent form.

Question	Scheme	Marks
3(a)	$\frac{dv}{dt} = -2(t+4)^{-\frac{1}{2}}$	M1
	$v = -\int 2(t+4)^{-\frac{1}{2}}\,dt$	
	$v = -4(t+4)^{\frac{1}{2}} \quad (+c)$	dM1 A1
	$t = 0, v = 8 \Rightarrow c = 16$	M1
	$v = 16 - 4(t+4)^{\frac{1}{2}}$ (m s^{-1}) *	A1 **cso**
		(5)
(b)	$v = 0 \quad 16 = 4(t+4)^{\frac{1}{2}}$	M1
	$16 = t + 4 \quad t = 12$	A1
	$x = 4\int\left(4 - (t+4)^{\frac{1}{2}}\right)dt$	
	$x = 4\left(4t - \frac{2}{3}(t+4)^{\frac{3}{2}}\right) \quad (+d)$	M1 A1
	$t = 0,\ x = 0 \quad d = 4\times\frac{2}{3}\times 4^{\frac{3}{2}} = \frac{64}{3}$ oe	A1
	$t = 12 \quad x = 4\left(4\times 12 - \frac{2}{3}\times 16^{\frac{3}{2}}\right) + \frac{64}{3} = 42\frac{2}{3}$ (m) oe eg 43 or better	dM1 A1
		(7)
		(12 marks)

Notes:

(a)

M1: Attempting an expression for the acceleration in the form $\frac{dv}{dt}$; minus may be omitted.

DM1: Attempting the integration

A1: Correct integration, constant of integration may be omitted (no ft)

M1: Using the initial conditions to obtain a value for the constant of integration

A1: **cso.** Substitute the value of c and obtain the final GIVEN answer

(b)

M1: Setting the **given** expression for v equal to 0

A1: Solving to get $t = 12$

M1: Setting $v = \frac{dx}{dt}$ and attempting the integration wrt t. At least one term must clearly be integrated.

A1: Correct integration, constant may be omitted.

Question 3 notes *continued*	
M1:	Substituting $t=0,\ x=0$ and obtaining the correct value of *d*. Any equivalent number, inc decimals.
dM1:	Substituting their value for *t* and obtaining a value for the required distance. Dependent on the second M mark.
A1:	Correct final answer, any equivalent form.

Question	Scheme	Marks
4(a)	Energy to top: $\frac{1}{2}\times 3m\times u^2-\frac{1}{2}\times 3mv^2=3mga$	M1 A1
	NL2 at top: $T+3mg=3m\frac{v^2}{a}$	M1 A1
	$T=3m\frac{u^2}{a}-6mg-3mg$	dM1
	$T\geqslant 0 \Rightarrow \frac{u^2}{a}\geqslant 3g$	M1
	$u^2\geqslant 3ag$	A1 cso
		(7)
(b)	Tension at bottom:	
	$\frac{1}{2}\times 3m\times V^2-\frac{1}{2}\times 3mu^2=3mga$	M1
	$T_{\max}-3mg=3m\frac{V^2}{a}$	M1
	$T_{\max}=3mg+6mg+3m\frac{u^2}{a}$	A1
	$T_{\min}=3m\frac{u^2}{a}-9mg$	
	$9mg+3m\frac{u^2}{a}=3\left(3m\frac{u^2}{a}-9mg\right)$	dM1
	$u^2=6ag$ *	A1 cso
		(5)
		(12 marks)

Notes:

(a)

M1: Attempting an energy equation, can be to a general point for this mark. Mass can be missing but use of $v^2=u^2+2as$ scores M0

A1: Correct equation from A to the top.

M1: Attempting an equation of motion along the radius at the top, acceleration in either form.

A1: Correct equation, acceleration in form $\frac{v^2}{r}$

dM1: Eliminate v^2 to obtain an expression for T dependent on both previous M marks.

M1: Use $T\geqslant 0$ at top to obtain an inequality connecting a, g and u

A1: Re-arrange to obtain the GIVEN answer.

Question 4 notes *continued*
(b) **M1:** Attempting an energy equation to the bottom, maybe from A or from the top. **M1:** Attempting an equation of motion along the radius at the bottom. **A1:** Correct expression for the max tension. **dM1:** Forming an equation connecting *their* tension at the top with *their* tension at the bottom. If the 3 is multiplying the wrong tension this mark can still be gained. Dependent on both previous M marks. **A1:** **cso.** Obtaining the GIVEN answer.

Question	Scheme	Marks
5(a)	$T=\frac{20e}{2}=\frac{15(1.8-e)}{1.2}$	M1A1
	$10e\times1.2=15(1.8-e)$	
	$e=1$	A1
	$AO=3\text{m}$ *	A1cso
		(4)
(b)	$0.5\ddot{x}=\frac{20(1-x)}{2}-\frac{15(0.8+x)}{1.2}$	M1 A1 A1
	$\ddot{x}=-45x \quad \therefore \text{SHM}$	A1 cso
		(4)
(c)	String becomes slack when $x=(-)0.8$ (allow wo sign due to symmetry)	B1
	$v^2=\omega^2(a^2-x^2)$	
	$v^2=45(1-0.8^2) \quad (=16.2)$	M1 A1 ft
	$v=4.024...$ m s^{-1} (4.0 or better)	A1ft
		(4)
(d)	$\frac{1}{2}\times\frac{20y^2}{2}-\frac{1}{2}\times\frac{20\times1.8^2}{2}=\frac{1}{2}\times0.5\times16.2$ ft on v	M1 A1 A1 ft
	$20y^2-64.8=16.2$	
	$y^2=4.05 \quad y=2.012....$	A1
	Distance $DB=\lvert 5-4.012...\rvert=0.988...\text{m}$ (accept 0.99 or better)	A1ft
	Alternative	
	$0.5\text{a}=-10(1.8+x)$	
	$v\frac{\text{d}v}{\text{d}x}=-36-10x$	
	$\int v\,\text{d}v=-\int(36+10x)\,\text{d}x$	
	$\frac{v^2}{2}=-36x+5x^2+c$	M1 A1
	$x=0,\ v=\frac{9\sqrt{5}}{5} \therefore c=8.1$	A1
	Then $v=0$ etc	M1 A1
		(5)
		(17 marks)

Question 5 *continued*
Notes:
(a) **M1:** Attempting to obtain and equate the tensions in the two parts of the string. **A1:** Correct equation, extension in *AP* or *BP* can be used or use *OA* as the unknown. **A1:** Obtaining the correct extension in either string (ext in *BP* =0.8 m) or another useful distance. **A1:** **cso.** Obtaining the correct GIVEN answer.
(b) **M1:** Forming an equation of motion at a general point. There must be a difference of tensions, both with the variable. May have *m* instead of 0.5 Accel can be *a*. **A1 A1:** Deduct 1 for each error, *m* or 0.5 allowed, acceleration to be $\ddot{x}$ now. **A1:** **cso** Correct equation in the required form, with a concluding statement; *m* or 0.5 allowed.
Question 5 notes *continued*
(c) **B1:** For $x = \pm 0.8$ Need not be shown explicitly. **M1:** Using $v^2 = \omega^2\left(a^2 - x^2\right)$ with *their* (numerical) ω and their *x* **A1ft:** Equation with correct numbers ft their ω **A1ft:** Correct value for *v* 2sf or better or exact.
(d) **M1:** Attempting an energy equation with 2 EPE terms and a KE term. **A1:** 2 correct terms may have $(1.8 + x)$ instead of *y*. **A1ft:** Completely correct equation, follow through their *v* from (c) **A1:** Correct value for distance travelled after *PB* became slack. $x = 0.21$ **A1ft:** Complete to the distance *DB*. Follow through their distance travelled after *PB* became slack.

Question	Scheme	Marks
6(a)	Vol $= \pi\int_0^2 \left(x^2+3\right)^2 \mathrm{d}x$	M1
	$= \pi\int_0^2 \left(x^4+6x^2+9\right)\mathrm{d}x$	
	$= \pi\left[\frac{1}{5}x^5+2x^3+9x\right]_0^2$	dM1 A1
	$= \frac{202}{5}\pi \text{ cm}^3$ *	A1
		(4)
(b)	$\pi\int_0^2 x\left(x^2+3\right)^2 \mathrm{d}x = \pi\int_0^2 \left(x^5+6x^3+9x\right)\mathrm{d}x$	M1
	$= \pi\left[\frac{1}{6}x^6+\frac{3}{2}x^4+\frac{9}{2}x^2\right]_0^2$	A1
	$= \frac{158}{3}\pi$ (Or by chain rule or substitution)	A1
	C of m $= \frac{158}{3}\times\frac{5}{202}$, $= 1.3036... = 1.30$ cm	M1 A1
		(5)
(c)	Mass ratio $\quad 2\times\frac{202}{5}\pi \quad \frac{1}{3}\pi\times7^2\times6 \quad \left(\frac{404}{5}+98\right)\pi$	B1
	Dist from V $\quad 6.7 \quad 4.5 \quad \overline{x}$	B1
	$\frac{404}{5}\times6.7+98\times4.5 = \left(\frac{404}{5}+98\right)\overline{x}$	M1 A1 ft
	$\overline{x} = \dfrac{\frac{404}{5}\times6.7+98\times4.5}{\left(\frac{404}{5}+98\right)} = 5.494.... = 5.5$ cm Accept 5.49 or better	A1
		(5)
(d)	$\tan\theta = \frac{6-\overline{x}}{7} = \frac{0.5058...}{7}$	M1
	$\alpha = \tan^{-1}\left(\frac{6}{7}\right) - \tan^{-1}\left(\frac{0.5058...}{7}\right) = 36.468...° = 36°$ or better	M1 A1
		(3)
		(17 marks)

Notes:

(a)

M1: Using $\pi\int y^2\mathrm{d}x$ with the equation of the curve, no limits needed

Question 6 notes *continued*	
dM1:	Integrating their expression for the volume.
A1:	Correct integration inc limits now.
A1:	Substituting the limits to obtain the GIVEN answer.
(b)	
M1:	Using $(\pi)\int xy^2\text{d}x$ with the equation of the curve, no limits needed, π can be omitted.
A1:	Correct integration, including limits; no substitution needed for this mark.
A1:	Correct substitution of limits.
M1:	Use of $\dfrac{\pi\int xy^2\text{d}x}{\pi\int y^2\text{d}x}$ with their $\pi\int xy^2\text{d}x$. π must be seen in both numerator and denominator or in neither.
A1:	**cso.** Correct answer. Must be 1.30
(c)	
B1:	Correct mass ratio.
B1:	Correct distances, from *V* or any other point, provided consistent.
M1:	Attempting a moments equation.
A1ft:	Correct equation, follow through their distances and mass ratio.
A1:	Correct distance from *V*
(d)	
M1:	Attempting the tan of an appropriate angle, numbers either way up.
M1:	Attempting to obtain the required angle.
A1:	Correct final answer 2sf or more.